

## Well testing in fractured media: flow dimensions and diagnostic plots

### Essais de puits dans les milieux fracturés: dimensions d'écoulement et relevés de diagnostic

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#### ABSTRACT

Hydraulic tests in heterogeneous media, particularly fractured media, are difficult to analyze because of the absence of radial flow. The theory of flow dimensions introduced by Barker in 1988 (*Water Resour. Res.* 24(10), 1988, 1796) provided a method of analyzing pumping (constant-rate) tests in non-radial systems, and this approach was later extended to constant-pressure tests. However, little use seems to be made of the flow-dimension approach to well-test analysis, perhaps because no easily applied method has been presented for determining, at the initial stage of an analysis, if such an approach would be productive. Depending on the distribution of heterogeneities within an aquifer, flow to a well may have almost any dimension (not limited to linear, radial, or spherical), or no constant dimension at all. Any well-test analytical solution requires that hydraulic properties be stable on some scale before those properties can be uniquely quantified. For each type of hydraulic test (constant-rate, constant-pressure, or slug/pulse), we suggest that a diagnostic plot of the scaled first or second derivative of the pressure or flow-rate response be created to determine, first, if a stable flow dimension was reached during the test and, second, what the value of that flow dimension is. If a stable flow dimension was reached, the scaled derivative will exhibit a constant value (scaled to be equal to the flow dimension). If the scaled derivative does not stabilize at a constant value, then no flow dimension can be specified and no unique hydraulic properties can be inferred analytically from the test. In all cases, the scale of testing must be appropriate to the scale of underlying interest.

#### RÉSUMÉ

Les essais hydrauliques dans les milieux hétérogènes, et en particulier les milieux fracturés sont difficiles à analyser, en raison de l'absence de flux radial. La théorie des dimensions d'écoulement introduite par Barker [1] en 1988 a fourni une méthode d'analyse des essais de pompage (à débit constant) dans les systèmes non-radiaux, et cette approche a été étendue ultérieurement aux essais à niveau constant. Cependant, cette approche de la dimension d'écoulement semble peu utilisée pour l'analyse des pompages d'essai, peut-être parce qu'aucune méthode facilement utilisable n'a été présentée pour déterminer, au préalable, si une telle approche serait opérante. Selon la distribution des hétérogénéités dans la couche aquifère, l'écoulement vers un puits peut avoir presque n'importe quelle dimension (de non limitée à linéaire, radiale, ou sphérique), ou aucune dimension constante du tout. Toute solution analytique d'un pompage d'essai exige que les propriétés hydrauliques soient stables à une certaine échelle pour que ces propriétés puissent être quantifiées sans ambiguïté. Pour chaque type d'essai hydraulique (débit constant, niveau constant, ou slug test/pulse test), nous proposons qu'un relevé diagnostic des dérivées première et seconde de la réponse en pression ou en débit soit établi pour déterminer, premièrement, si une dimension d'écoulement stable a été atteinte pendant l'essai, et, deuxièmement, quelle est la valeur de cette dimension d'écoulement. Si une dimension d'écoulement stable a été atteinte, la dérivée mise à l'échelle présentera une valeur constante (mise à l'échelle de la dimension de l'écoulement). Si la dérivée mise à l'échelle ne se stabilise pas à une valeur constante, alors aucune dimension d'écoulement ne peut être spécifiée et aucune propriété hydraulique unique ne peut être déduite analytiquement de l'essai. Dans tous les cas, l'échelle de l'essai doit être appropriée à l'échelle de l'intérêt sous-jacent.

*Keywords:* Flow dimensions, hydraulic tests, diagnostic plots, heterogeneity, fractured media.

## 1 Introduction

Hydraulic tests in heterogeneous media, particularly fractured media, are notoriously difficult to analyze. One reason for this difficulty is that most well-test analysis techniques were derived for conditions in which flow is radially symmetric around a well, which occurs only under homogeneous conditions. In highly heterogeneous media, pressure transients propagating from well

tests encounter different hydraulic properties with both distance and direction from the source well, causing flow to be distinctly nonradial. The resulting pressure responses often fail to display the horizontal derivatives (or semilog straight lines) on which radial well-test interpretations rely.

In 1988, Barker introduced the theory of flow dimensions, providing a method of analyzing pumping (constant-rate) tests in non-radial systems. This approach was later extended to

constant-pressure tests (Doe, 1991). However, except for a few nuclear waste repository programs around the world (e.g. Noy *et al.*, 1998; Geier *et al.*, 1996), little use seems to be made of the flow-dimension approach to well-test analysis. One factor contributing to the apparent lack of interest in flow dimensions may be that no easily applied method has been presented for determining, at the initial stage of an analysis, if a flow-dimension approach is needed (i.e. flow is not radial). In addition, any well-test analytical solution requires that hydraulic properties be stable on some scale before those properties can be uniquely quantified. Under conditions of extreme heterogeneity, no stable flow dimension may exist and hydraulic tests conducted in such a medium may, in fact, be uninterpretable. We suggest a diagnostic approach to determine if the requisite flow dimension for a successful analysis exists that also provides the value of that dimension.

## 2 Flow dimensions

For those trained to think in terms of radial flow, flow dimensions can initially seem unnatural or a mere mathematical construct. One way to approach the subject is to recognize that all well tests, including those exhibiting radial flow, take place in a three-dimensional world. That being so, why don't all well tests exhibit spherical (i.e. three-dimensional) flow? The reason is quite simple: various factors act to limit the propagation of pressure transients in different directions. For radial flow to be observed, a system must be bounded by two parallel boundaries, usually (but not necessarily) an upper boundary and a lower boundary. These boundaries are nothing more than heterogeneity. Standard radial-flow models (e.g. Theis, 1935) assume that this heterogeneity has a "binary" character: the aquifer has a permeability of "1" and the overlying and underlying aquitards have a permeability of "0". Theories of leaky aquifers (e.g. Hantush, 1956) recognize that the heterogeneity might not be so drastic and that a binary approach is not adequate. Whether they assume fully confined or leaky conditions, however, the radial-flow models only consider heterogeneity at the boundaries of the aquifer, not within the aquifer itself.

Ferris *et al.* (1962), among others, considered the simple case in which a confined aquifer is truncated by a boundary such as a fault, again a form of binary heterogeneity. They found that the equations for radial flow could still be used to describe a well-test response in such a system, by superimposing an imaginary image-well response on the real well response. The petroleum industry, in particular, has expanded on this technique, developing models for most conceivable combinations of boundaries and geometries. With few exceptions, however, they all assume binary heterogeneity, which allows the continued use of radial-flow models.

But what happens when heterogeneous properties are spatially distributed throughout the aquifer itself and cannot be represented by a plane (or set of planes) with permeability of 0 on one side and 1 on the other? Examples might include a fractured medium with several fracture sets with different average permeabilities and preferred orientations, or a sedimentary environment with a

complex array of facies having different permeabilities. This is where Barker's (1988) theory of flow dimensions may apply.

Barker (1988) discussed flow systems where the flow dimension ( $n$ ) of the system was related to the power by which the flow area changed with distance from the source. Because he assumed constant hydraulic conductivity and specific storage,  $n$  described the geometry of the system. Barker (1988) derived the "generalized radial flow" equation to describe flow that occurs radially toward or away from a well in a homogeneous, isotropic aquifer and fills an  $n$ -dimensional space:

$$S_s \frac{\partial h}{\partial t} = \frac{K}{r^{n-1}} \frac{\partial}{\partial r} \left( r^{n-1} \frac{\partial h}{\partial r} \right) \quad (1)$$

where  $S_s$  = specific storage,  $1/L$ ;  $h$  = hydraulic head,  $L$ ;  $t$  = elapsed time,  $T$ ;  $K$  = hydraulic conductivity,  $L/T$ ;  $r$  = radial distance from borehole,  $L$ ;  $n$  = flow dimension.

The flow area ( $A$ ) in Barker's (1988) formulation is given by:

$$A(r) = b^{3-n} \frac{2\pi^{n/2}}{\Gamma(n/2)} r^{n-1} \quad (2)$$

where  $b$  = extent of the flow zone,  $L$ ;  $\Gamma$  = gamma function.

The flow dimension  $n$  is related to the power-law relationship between flow area and radial distance from the borehole. The flow dimension is defined as the power of variation plus one, as follows:

$$n = \frac{d \log A}{d \log r} + 1 \quad (3)$$

For example, the relationship between flow area and distance in a standard radial system is given by:

$$A(r) = 2\pi r b \quad (4)$$

The flow area is seen to vary linearly with distance ( $r^1$ ), making the flow dimension, by definition, two. All of the diagnostic methods used to deduce radial flow, i.e. the shapes of various type curves, depend only on the relationship between flow area and distance ( $r$ ). The shapes of the type curves are independent of the constant  $2\pi b$ , so they provide no information on the degree to which the flow system fills the available space.

In a heterogeneous medium, permeable features such as fractures may not be distributed in a space-filling manner that would lead to apparent radial flow. They could be arranged in a pattern that caused flow area to change nonlinearly with distance from any point, or flow could be channelized within high-aperture portions of the fractures to create the same effect. Thus, non-radial flow conditions can easily be created, even in a "two-dimensional" medium bounded above and below by aquitards. The key question has to do with whether heterogeneities are distributed in a manner that would cause flow area to increase as some consistent power of distance or if the distribution is too random for any consistent flow dimension to exist.

## 3 System geometry and hydraulic properties

All well-test-analysis methods require that the geometry of the system be specified (by specifying the flow dimension), and then

provide estimates of the hydraulic properties *given that geometry*. For instance, the standard Theis (1935) solution rests on an assumption of radial flow and provides correct parameter estimates only when that assumption is met. If a constant geometry cannot be specified, that is, if there is no volume over which the flow dimension does not change, then no unique hydraulic properties can be inferred analytically.

Thus, analyses of well tests in heterogeneous media can fail for either of two reasons: (1) the system geometry is not constant (no stable flow dimension), or (2) a stable flow dimension exists, but has been misspecified (most commonly mistakenly assumed to be radial). Determining whether or not a stable flow dimension is present should, therefore, be one of the first steps in any well-test analysis. We propose a method for simultaneously determining, first, if a stable flow dimension exists, and second, if so, what that dimension is.

#### 4 Flow-dimension diagnostic plots

Roberts *et al.* (1999) developed what they term “flow-dimension diagnostic plots” for each of the three common types of hydraulic tests (constant-rate, constant-pressure, and slug/pulse). These diagnostic plots display a scaled first or second derivative of the pressure or flow-rate response. They can be used to determine, first, if a stable flow dimension was encountered during the test and, second, what that dimension is. If a stable flow dimension exists, the scaled derivative will exhibit a constant value (scaled to be equal to the flow dimension). We suggest that this constant value persist for at least one log cycle of time to have confidence in it. If the scaled derivative does not stabilize at a constant value, then no stable flow dimension exists (on the scale of the test) and no unique hydraulic properties can be inferred analytically from the test. (This does not preclude the use of numerical non-linear regression techniques for test interpretation.)

#### 4.1 Constant-rate tests

The standard diagnostic plot for a constant-rate test is a log–log plot of elapsed time on the  $x$ -axis versus the pressure change and derivative of pressure change with respect to the natural log( $\ln$ ) of time (or superposition time) on the  $y$ -axis (Ehlig-Economides *et al.*, 1990; Horne, 1995). A variety of different well, aquifer, and boundary conditions can be identified simply from the shapes of the curves on this graph (Fig. 1).

As noted by Ehlig-Economides *et al.* (1990), among others, the pressure derivative ( $p'$ ) developed by Bourdet *et al.* (1989) displays late-time straight lines with slopes related to the flow geometry/dimension. The relationship between the slope of the pressure derivative ( $m$ ) and the flow geometry/dimension ( $n$ ) is given by Barker (1988) as:

$$m = 1 - \frac{n}{2} \quad (5)$$

Defining a scaled second derivative as:

$$p'' = -2 \frac{d \log(p')}{d \log(t)} + 2 \quad (6)$$

Roberts *et al.* (1999) created a semilog plot of the scaled second derivative versus log time such that the late-time data will plot as a constant value equal to  $n$ . Figure 2 shows an example flow-dimension diagnostic plot for a constant-rate test with pumping-well data from simulated tests in systems of various flow dimensions. A flow dimension of 1.2 might be found in a fractured system with a single preferred fracture orientation, so that system behavior was close to linear. A flow dimension of 2.7 might be found from a test conducted at a point in a crystalline rock mass with fractures distributed nearly evenly throughout three dimensions. The transforms described above can also be applied to the observation-well data from a constant-rate test to create flow-dimension diagnostic plots.

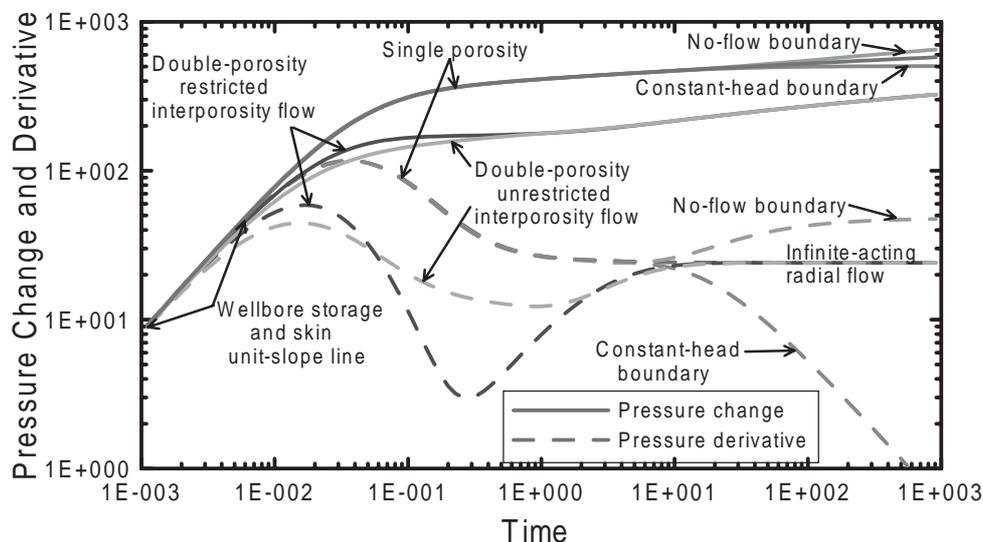


Figure 1 Log–log diagnostic plot for constant-rate tests.

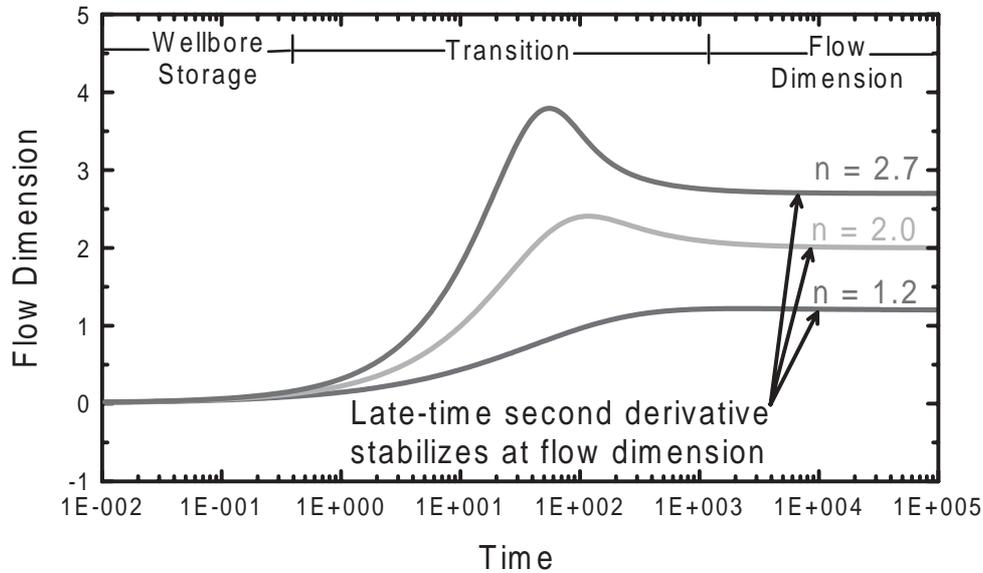


Figure 2 Flow-dimension diagnostic plot for constant-rate tests with  $n = 1.2, 2.0,$  and  $2.7$ .

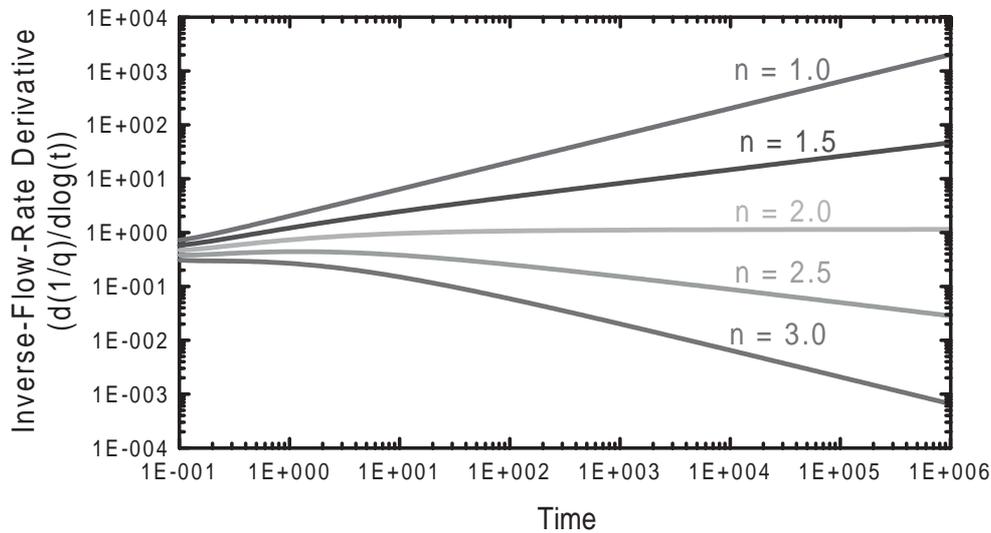


Figure 3 Derivative of  $1/q$  for a constant-pressure test for selected values of  $n$ .

Note that the scaled second derivative cannot be used to diagnose flow dimension visually during the periods in which the pressure response is dominated by factors other than the hydraulic response of the tested formation (such as wellbore storage), i.e. before the first derivative takes on a constant slope. This is not to say that a response cannot be matched by an analytical model during the period before derivative stabilization has occurred, but that such a match depends on the assumption of a flow dimension that is not in evidence. The match is likely, therefore, to be non-unique.

#### 4.2 Constant-pressure tests

The standard plotting format for a constant-pressure test is a log-log plot of elapsed time on the  $x$ -axis versus the flow rate ( $q$ ) on the  $y$ -axis (Jacob and Lohman, 1952). Late-time flow-rate data for all  $n < 2$  plot as a straight line on this type of plot, with a unique slope for each value of  $n$ . Flow-rate data also plot as a straight line for all  $n > 2$ , but the slope is not unique, being zero

for all  $n > 2$ . No straight line develops for  $n = 2$ . Consequently, this way of presenting the data does not lead to a productive method for determining flow dimensions in all cases.

A more useful type of constant-pressure plot displays a straight-line behavior for all values of  $n$ . For this type of plot, an inverse-flow-rate derivative is calculated as:

$$\left(\frac{1}{q}\right)' = \frac{d(1/q)}{d \log(t)} \quad (7)$$

and plotted versus elapsed time on a log-log plot (Geier *et al.*, 1996) (Fig. 3).

The late-time data for all values of  $n$  will exhibit straight lines whose slopes are related to  $n$  by:

$$m = 1 - \frac{n}{2} \quad (8)$$

The flow dimension can be estimated from the slope of the straight line. If the data are relatively free of noise, a scaled second

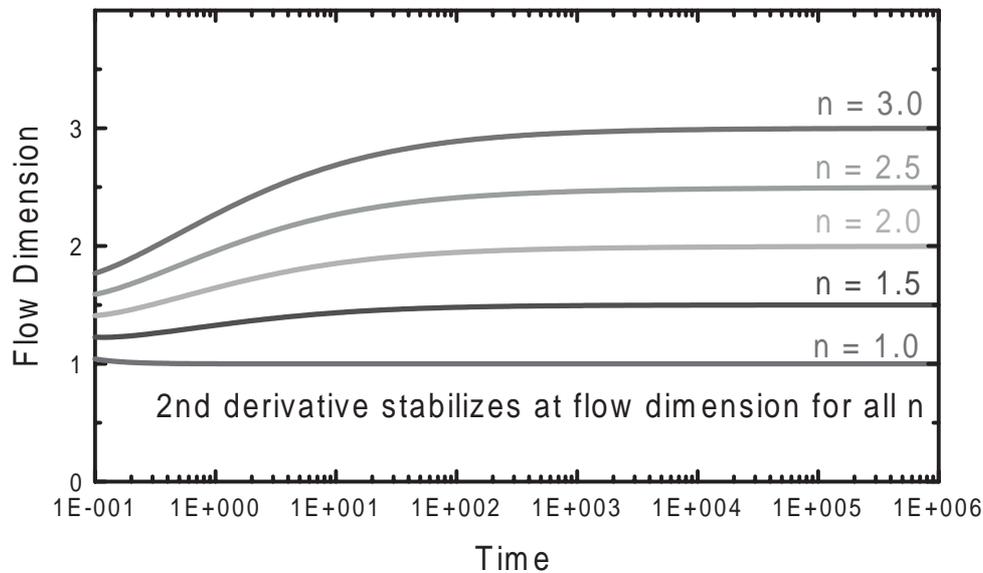


Figure 4 Flow-dimension diagnostic plot for constant-pressure tests using  $1/q$ .

derivative of  $1/q$  can be calculated as:

$$\left(\frac{1}{q}\right)'' = -2 \frac{d \log(1/q)'}{d \log(t)} + 2 \quad (9)$$

and plotted versus log time on a semilog plot (Fig. 4) and the flow dimension read directly.

The transforms to create flow-dimension diagnostic plots from constant-pressure test observation-well data (pressure) are the same as those applied to the transient flow-rate data from the test well—simply substitute pressures for flow rates ( $q$ ) in (7) and (9).

#### 4.3 Slug/pulse tests

Cooper *et al.* (1967) developed the analytical approach to slug tests that has formed the basis for most other researchers' work. Bredehoeft and Papadopulos (1980) extended the approach to slug tests conducted under shut-in conditions, generally referred to in the groundwater literature as pulse tests. Both sets of authors presented their solutions as normalized pressure change versus log time on semilog plots. Ramey *et al.* (1975) derived an analogous solution for the petroleum industry incorporating wellbore storage and skin. In addition to the semilog plotting format originated by Cooper *et al.* (1967), Ramey *et al.* (1975) plotted their results in log-log format as both normalized pressure change versus time and one minus the normalized pressure change versus time. In the petroleum literature, these plots are referred to as the Ramey A, B, and C plots, respectively. We take the Ramey B log-log plot of normalized pressure change versus time as our starting point.

To begin, the slug- or pulse-test data are converted to a normalized response using the following transform:

$$P_{\text{norm}} = \frac{p_i - p_t}{p_i - p_0} \quad (10)$$

where  $p_{\text{norm}}$  = normalized pressure;  $p_i$  = initial pressure before slug or pulse began,  $M/LT^2$ ;  $p_t$  = pressure at time  $t$ ,  $M/LT^2$ ;  $p_0$  = pressure at time  $t_0$  (slug/pulse maximum),  $M/LT^2$ .

On a log-log plot (Fig. 5), the late-time data plot as straight lines with slope ( $m$ ) related to the flow dimension ( $n$ ) of the system by:

$$m = \frac{-n}{2} \quad (11)$$

Given this relationship between  $m$  and  $n$ , the log-log derivative of the normalized response can be scaled such that it will stabilize at a constant value equal to the dimension of the system. The scaled derivative is given by:

$$P'_{\text{norm}} = -2 \frac{d \log(p_{\text{norm}})}{d \log(t)} \quad (12)$$

Figure 6 shows typical scaled derivatives for  $n = 1, 2,$  and  $3$ .

Flow-dimension diagnostic plots for slug/pulse observation wells are created in the same manner as those for the test well. Pressure is normalized using (10), where  $p_0$  is the slug/pulse maximum at the observation well, and the diagnostic derivative is calculated using (12).

## 5 Examples

A pumping test conducted in a fractured dolomite provides an example of a fractured medium exhibiting a stable flow dimension. Figure 7 shows the log-log diagnostic plot for this test. The late-time pressure derivative is not horizontal as it would be in a radial flow system, but instead has a positive log-log slope. The flow-dimension diagnostic plot is shown in Fig. 8, and shows clear indications of a stable flow dimension between approximately 10 and 300 elapsed hours with a flow dimension of approximately 1.2. Thus, this test is interpretable.

A second example is provided by a slug test conducted in another well in the same fractured dolomite, approximately 3 km from the pumping test well. Figure 9 shows the log-log Ramey B plot of the data from this test. The flow-dimension diagnostic

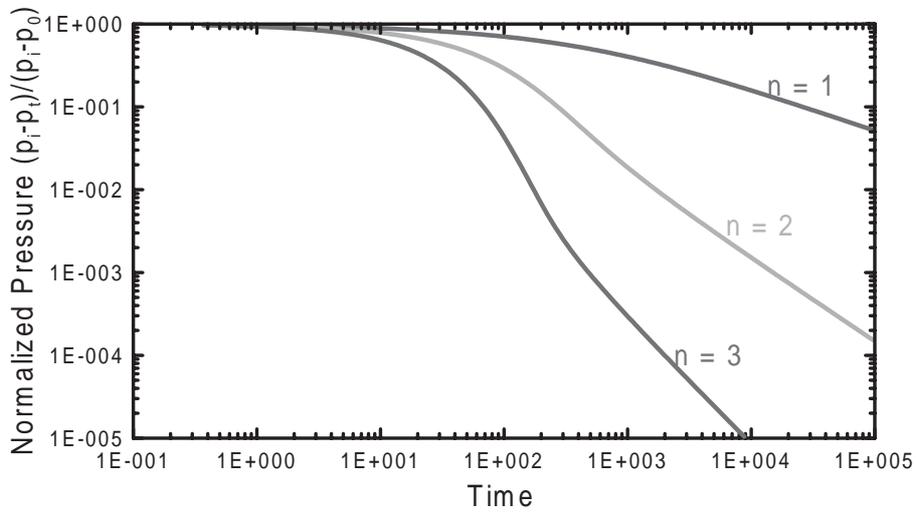


Figure 5 Ramey B log-log plot of slug/pulse-test data for  $n = 1, 2,$  and  $3.$

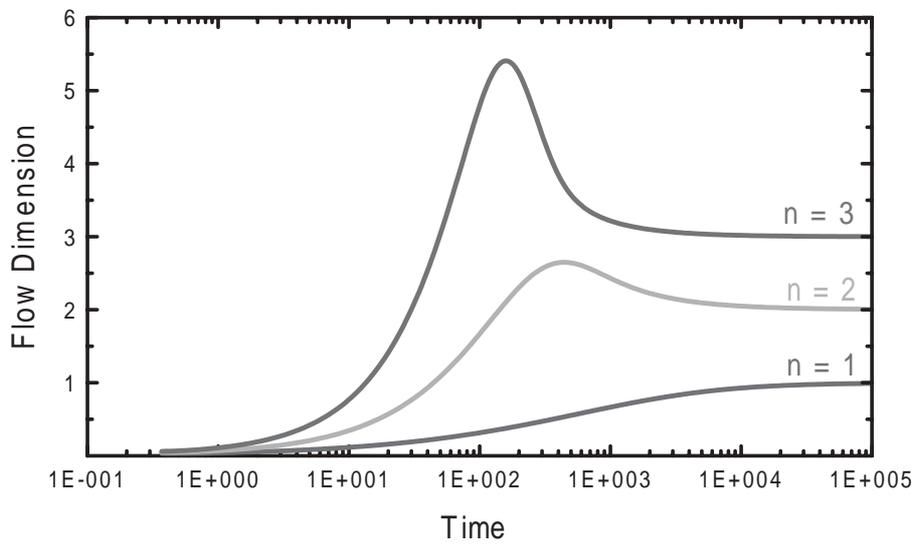


Figure 6 Flow-dimension diagnostic plot for slug/pulse tests for  $n = 1, 2,$  and  $3.$

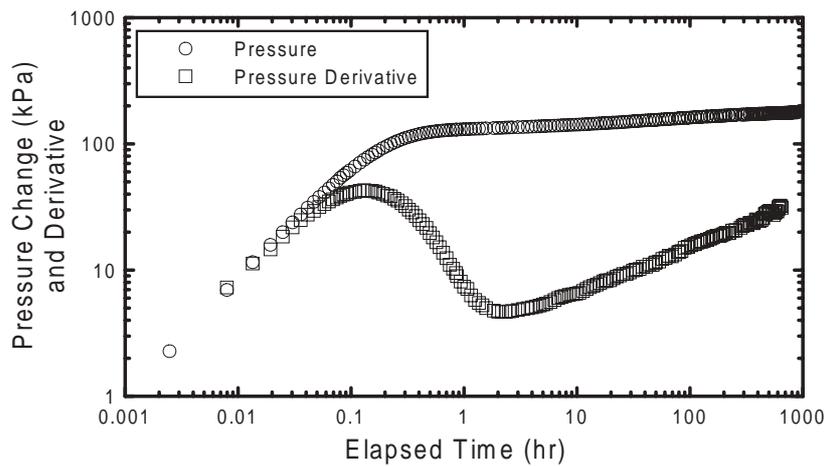


Figure 7 Log-log plot of pumping test data.

plot (Fig. 10) shows the late-time derivative oscillating between approximately 0.8 and 1.2, without stabilizing at a constant value. Hence, no stable flow dimension appears to have been reached in this test.

The differences between these tests are informative. The 1.6-h slug test interrogated a much smaller volume of rock than the

several hundred hours pumping test, and failed to reach a stable flow dimension. For the pumping test, on the other hand, a stable flow dimension was reached after approximately 10 h. If the scale of underlying interest is close to that of the slug test, then well testing may be futile; heterogeneity on this scale cannot be represented by a single dimension. If the scale

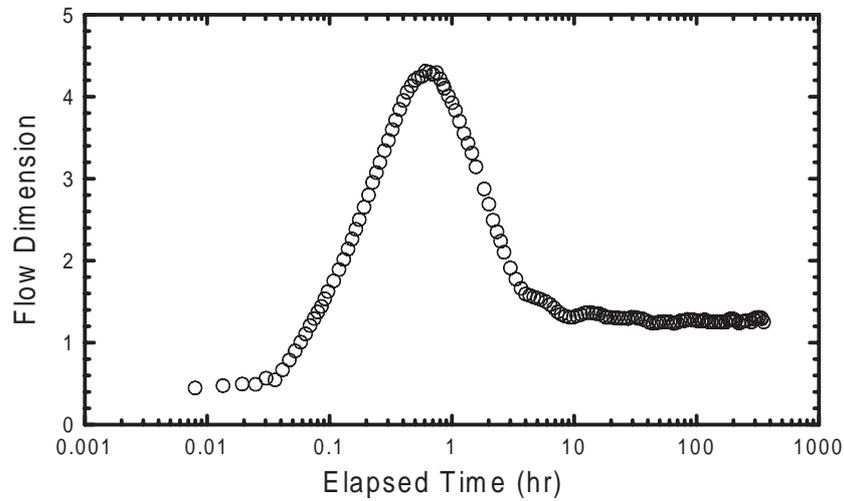


Figure 8 Flow-dimension diagnostic plot of pumping test data showing a flow dimension of 1.2.

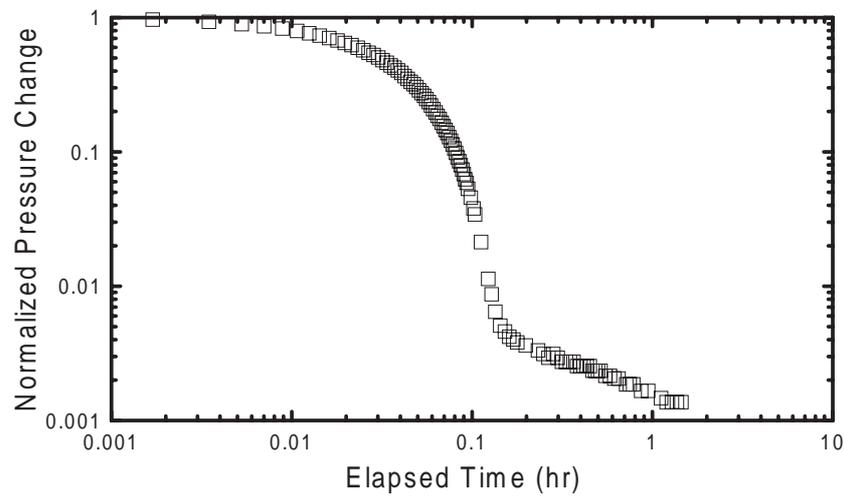


Figure 9 Log-log (Ramey B) plot of slug test.

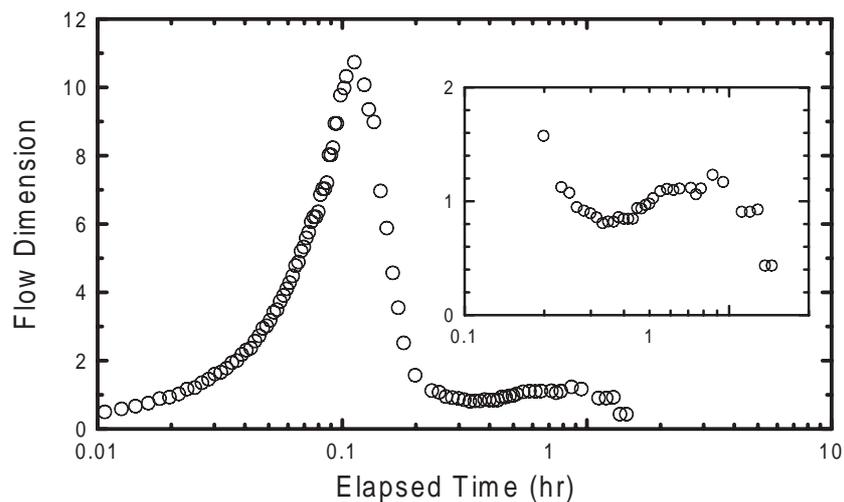


Figure 10 Flow-dimension diagnostic plot for slug test showing no clear flow dimension.

of interest is larger, then pumping tests might provide useful information.

## 6 Limitations

Construction and use of flow-dimension diagnostic plots is not without limitations. First, data must be collected for a sufficient

period of time for the flow dimension to become evident. While this should not present an undue burden for constant-rate or constant-pressure tests, slug or pulse tests may need to be run until greater than 95% or even 99% recovery has occurred, which is somewhat uncommon in practice. Late-time slug- and especially pulse-test data, on which flow dimension definition depends, are also susceptible to undesirable superposition effects

if the pressure was not truly static before the test began. Second, as the flow-dimension diagnostic plots rely on first or second derivatives, the data must be of high quality. Modern pressure transducers typically provide data of adequate quality, provided that barometric and tidal effects are minimal or can be effectively filtered out. Data quality also depends on holding rates and pressures constant during constant-rate and constant-pressure tests, respectively. Finally, flow-dimension diagnostic plots cannot be created for the recovery (buildup) periods following constant-rate tests. The standard diagnostic plots and pressure derivatives widely used for interpretation of recovery data rely on linear superposition of the recovery response on the drawdown response (Bourdet *et al.*, 1989). Linear superposition is not valid for any flow dimension other than 2. Under any of these circumstances, the inability to develop a useful flow-dimension diagnostic plot in no way precludes the use of numerical non-linear regression techniques to analyze a test successfully.

## 7 Summary and conclusions

Flow-dimension diagnostic plots provide a simple and effective means of evaluating, first, if a stable flow dimension exists in a heterogeneous medium and, second, what that flow dimension is. All well-test analytical solutions require that the geometry of the system be specified (by specifying the flow dimension), and then provide estimates of the hydraulic properties *given that geometry*. If a constant geometry cannot be specified, that is, if there is no volume over which the flow dimension does not change, then no unique hydraulic properties can be inferred analytically.

The examples given in this paper highlight the importance of scale. If a stable flow dimension is present in a heterogeneous system at all, the method of testing must be appropriate to assess the scale at which the flow dimension exists. Thus, a slug test, which typically has a smaller radius of influence than a pumping test, may not necessarily reveal the flow dimension shown by a pumping test if that dimension does not exist at the smaller scale. Conversely, if a flow dimension has an upper limit on its size, a slug test might show it while a pumping test might not. Thus, two factors need to be balanced in any investigation: the scale of interest and the scale of testing. Testing should always be performed at the same scale as the features or processes of underlying interest. Information obtained on flow dimension at one scale cannot be assumed to be valid at a different scale. As this method gains wider acceptance and use, the technical community will be able to see if stable flow dimensions commonly, or rarely, occur in fractured and heterogeneous media, which should have implications for future hydraulic-testing strategies.

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